

Top 10 Questions

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अतिलघु उत्तरीय प्रश्न

✓ 1. सिद्ध कीजिए-

✓ $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$, $x \in [-1, 1]$

(इल)

माना $\cos^{-1}(-n) = A$

$-n = \cos A$

$n = -\cos A$

$n = \cos(180 - A)$

$n = \cos(\pi - A)$

$\cos^{-1} n = \pi - A$

$\cos^{-1} n = \pi - \cos^{-1} n$

$\boxed{\cos^{-1}(-n) = \pi - \cos^{-1}(n)}$

(2019)

sin, cos ~

+ All positive

✓

tan
cot

sec
cosec

-

✓ 2. $\int \frac{dx}{(e^x - 1)}$ का मान ज्ञात कीजिए।
(2009, 13, 15, 17)

सलू. $I = \int \frac{dn}{(e^n - 1)}$

$$= \int \frac{dn}{e^n \left(1 - \frac{1}{e^n}\right)}$$

$$= \int \frac{e^{-n} dn}{\left(1 - e^{-n}\right)}$$

$$= \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \log(1 - e^{-n}) + C$$

माना $(1 - e^{-n}) = t$

$\circ \frac{d}{dn}(-e^{-n}) = dt$
 $-e^{-n} \cdot (-1) dn = dt$
 $e^{-n} dn = dt$

3. $\int x^2 \sin(x^3) dx$ का मान ज्ञात कीजिए।

(2018)

$$\begin{aligned} I &= \int n^2 \sin(n^3) dn \\ &= \int \frac{1}{3} \cdot \sin t dt & \text{माना } n^3 = t & \frac{d}{dn}(n^3) = n^2 \cancel{n^{-1}} \\ &= \frac{1}{3} \int \sin t dt & \frac{d}{dt}(n^3) = dt & 3n^2 dn = dt \\ &= \frac{1}{3} [-\cos t + C] & 3n^2 dn = dt \\ &= \frac{1}{3} [-\cos n^3] + C & n^2 dn = \underline{\underline{\frac{dt}{3}}} \end{aligned}$$

✓
4. $\int \frac{\sec x}{\sec x + \tan x} dx$ का मान ज्ञात
कीजिए।

(2018, 19)

$$\begin{aligned} \text{Sol} \quad I &= \int \frac{\sec n}{\sec n + \tan n} dn \\ &= \int \frac{\sec n}{(\sec n + \tan n)(\sec n - \tan n)} (\sec n - \tan n) dn & (a+b)(a-b) = a^2 - b^2 \\ &= \int \sec^2 n - \sec n \cdot \tan n dn \end{aligned}$$

$$\begin{aligned}
 & \int (\sec n + \tan n)(\sec n - \tan n) dn \\
 &= \int \frac{\sec^2 n - \sec n \cdot \tan n}{\sec^2 n - \tan^2 n} dn \\
 &= \int \frac{\sec^2 n - \sec n \cdot \tan n}{1} dn \\
 &= \int \sec^2 n dn - \int \underline{\sec n \cdot \tan n} dn \\
 &= \tan n - \sec n + C
 \end{aligned}$$

$$\begin{aligned}
 \text{if } \tan^2 n &= \sec^2 n \\
 1 &= \sec^2 n - \tan^2 n
 \end{aligned}$$

5. यदि $y = \tan^{-1} \left\{ \frac{2x}{1-x^2} \right\}$, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

(अथवा) यदि $y = \tan^{-1} \left\{ \frac{2x}{1-x^2} \right\}$ तो दिखाइए कि $\frac{dy}{dx} = \frac{2}{1+x^2}$.

(2009, 11, 12, 13)

(2015, 16)

(Ex) $y = \tan^{-1} \left\{ \frac{2n}{1-n^2} \right\}$

माना $n = \tan \theta$
 $\tan^{-1} n = \theta$

$y = \tan^{-1} \left(\frac{2 \tan \theta}{1-\tan^2 \theta} \right)$

$y = \tan^{-1} (\tan 2\theta)$

$y = 2\theta$

$y = 2 \tan^{-1} n$

diff w.r.t n

diff w.r.t n

$$\frac{dy}{dn} = 2 \times \frac{1}{1+n^2}$$

$$\frac{d}{dn} \tan^{-1} n = \frac{1}{1+n^2}$$

$$\frac{dy}{dn} = \frac{2}{1+n^2}$$



6. $\sin^{-1} \left(\sin \frac{7\pi}{4} \right)$ का मुख्य मान ज्ञात

कीजिए।

(2020)

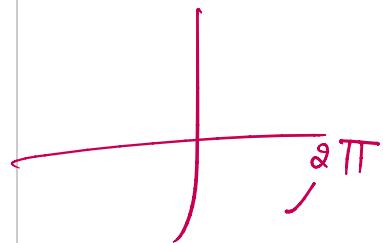
माना $\sin^{-1} \left(\sin \frac{7\pi}{4} \right) = n$

$$\sin n = \sin \frac{7\pi}{4} \quad / \quad -\frac{\pi}{2} \leq n \leq \frac{\pi}{2}$$

$$\sin n = \sin \left(2\pi - \frac{\pi}{4} \right)$$

$$\sin n = \sin \left(-\frac{\pi}{4} \right)$$

$$n = -\frac{\pi}{4}$$



✓ 7. यदि $A = \begin{bmatrix} \frac{2+i}{3} & \frac{-i}{4i} \\ \frac{3}{2i} & \frac{1+i}{3} \end{bmatrix}$ तथा $B = \begin{bmatrix} \frac{1+i}{2i} & \frac{2i}{3} \\ \frac{2i}{3} & \frac{3}{2i} \end{bmatrix}$
हो तो $A + B$ का मान बताइए। (2017, 18)

$$\begin{aligned}
 A + B &= \begin{bmatrix} 2+i & -i \\ 3 & ui \end{bmatrix} + \begin{bmatrix} 1+i & 2i \\ 2i & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 2+i+1+i & -i+2i \\ 3+2i & ui+3 \end{bmatrix} \\
 &= \begin{bmatrix} 3+2i & i \\ 3+2i & ui+3 \end{bmatrix}
 \end{aligned}$$

✓ 8. यदि आव्यूह $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ तथा आव्यूह $B = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$, तो सिद्ध कीजिए कि $AB = 2B$. (2011, 12, 14)

$$\begin{aligned}
 AB &= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \\
 &= \begin{bmatrix} 2x_1 + 0x_2 + 0x_3 & 2y_1 + 0y_2 + 0y_3 & 2z_1 + 0z_2 + 0z_3 \\ 0x_1 + 2x_2 + 0x_3 & 0y_1 + 2y_2 + 0y_3 & 0z_1 + 2z_2 + 0z_3 \\ 0x_1 + 0x_2 + 2x_3 & 0y_1 + 0y_2 + 2y_3 & 0z_1 + 0z_2 + 2z_3 \end{bmatrix} \\
 &= \begin{bmatrix} 2x_1 & 2y_1 & 2z_1 \\ 2x_2 & 2y_2 & 2z_2 \\ 2x_3 & 2y_3 & 2z_3 \end{bmatrix} \\
 &= 2 \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = 2B
 \end{aligned}$$

✓ 9. x, y तथा z का मान बताइए, यदि $\begin{bmatrix} 3 & x \\ 4 & y \end{bmatrix} = 2 \times \begin{bmatrix} 1.5 & 1 \\ z & 1 \end{bmatrix}$. (2013, 15)

(*) $\begin{bmatrix} 3 & x \\ 4 & y \end{bmatrix} = 2 \times \begin{bmatrix} 1.5 & 1 \\ z & 1 \end{bmatrix}$

$$\begin{bmatrix} 3 & x \\ 4 & y \end{bmatrix} = \begin{bmatrix} 3 \cdot 0 & 2 \\ 2z & 2 \end{bmatrix}$$

$$\begin{bmatrix} s & 1 \\ u & y \end{bmatrix} = \begin{bmatrix} s & 0 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & n \\ u & y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$$

$$n=2, \quad y=2, \quad , \quad \begin{array}{l} 2 \\ 2 = 2 \end{array}$$

$c_1 \quad c_2 \quad c_3$

✓ 10. सिद्ध कीजिए कि $\begin{vmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{vmatrix} = 0.$

(2015, 16, 20)

$$c_1 \rightarrow c_1 + c_3$$

$$\left| \begin{array}{ccc} 32 & 16 & 19 \\ 34 & 17 & 20 \\ 36 & 18 & 21 \end{array} \right|$$

$$c_1 \rightarrow c_1 - 2c_2$$

$$\left| \begin{array}{ccc} 0 & 16 & 19 \\ 0 & 17 & 20 \\ 0 & 18 & 21 \end{array} \right|$$

$$\underline{\underline{=0}}$$

2

Q1. $e^x \log_a x$ का x के सापेक्ष अवकल गुणांक

ज्ञात कीजिए।

(2011, 15, 17)

$$y = c^n \left(\log_a n \right)$$

$$\log_a m = \frac{\log_b m}{\log_b a}$$

$$e^n \log_e n \cdot \log_e e$$

$$\log_a n = \log_e n \log_a e$$

$$\frac{dy}{dn} = \frac{d}{dn} (e^n \log_e^n \cdot \log_5)$$

$$= \log_e \frac{d}{dn} (e^n \cdot \underline{\log_e^n})$$

$$= \log_a \left(e^n \times \frac{1}{n} + \log_e x e^n \right)$$

$$= \log_a \left(\frac{e^n}{n} + e^n \times \log_e^n \right)$$

$$= e^n \left[\frac{\log_e}{n} + \frac{\log_e \times \log_e n}{n} \right]$$

$$= e^n \left[\frac{\log_a e}{n} + \overline{\log_a n} \right]$$

✓ 32. यदि $y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$, तो दिखाइए

कि $(1+x^2) \frac{dy}{dx} - 2 = 0$. ✓ ✓
(2014, 18)

(अथवा) यदि $y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$, तो $\frac{dy}{dx}$ का

मान ज्ञात कीजिए।

(2020)

$$y = \cos^{-1} \left(\frac{1-n^2}{1+n^2} \right)$$

माना $n = \tan \theta$

$$y = \cos^{-1} \left(\frac{1-\tan^2 \theta}{1+\tan^2 \theta} \right)$$

$$y = \cos^{-1} (\cos 2\theta)$$

$$y = 2\theta$$

$$y = 2 \tan^{-1} n$$

diff wrt θ (n)

$$\frac{dy}{dn} = 2 \frac{d}{dn} \tan^{-1} n \quad \frac{d}{dn} \tan^{-1} n = \frac{1}{1+n^2}$$

$$\frac{dy}{dn} = \frac{2}{1+n^2}$$

$$(1+n^2) \frac{dy}{dn} = 2, \quad \left((1+n^2) \frac{dy}{dn} \right) - 2 = 0$$

