लघु उत्तरीय प्रश्न

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1. यदि $A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$ तथा $B = \begin{bmatrix} 5 & 7 \\ 2 & 0 \end{bmatrix}$, तो सिद्ध कीजिए कि $(AB)' = B'A'$. (2012, 14, 17)

$A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 7 \\ 2 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 10+10 & 14+0 \\ 15+2 & 21+0 \end{bmatrix} = \begin{bmatrix} 20 & 14 \\ 17 & 21 \end{bmatrix}$

$(AB)' = \begin{bmatrix} 20 & 17 \\ 14 & 21 \end{bmatrix}$ ✓

$B' = \begin{bmatrix} 5 & 2 \\ 7 & 0 \end{bmatrix}$, $A' = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$

$B'A' = \begin{bmatrix} 5 & 2 \\ 7 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$

$= \begin{bmatrix} 10+10 & 15+2 \\ 14+0 & 21 \end{bmatrix}$

$= \begin{bmatrix} 20 & 17 \\ 14 & 21 \end{bmatrix}$ ✓

2. यदि $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ तथा

2. यदि $X+Y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$ तथा $X-Y=\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$ है तो X तथा Y का मान ज्ञात कीजिए।
(2019, 20)

$$X+Y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$$

$$X-Y=\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$$

$$X+Y+X-Y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}+\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$$

$$2X=\begin{bmatrix}8 & 8\\ 0 & 8\end{bmatrix}$$

$$X=\begin{bmatrix}4 & 4\\ 0 & 4\end{bmatrix}$$

$$X+Y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$$

$$Y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}-\begin{bmatrix}4 & 4\\ 0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$$

3. यदि $[x \ -5 \ -1]\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=0$, तो x का मान ज्ञात कीजिए। (2016, 17, 18)

$$\begin{bmatrix}x-2\\ -10\\ 2x-5-3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=0$$

$$[(x-2)x-40+(2x-8)1]=0$$

$$x^2-2x-40+2x-8=0$$

$$x^2=48$$

$$x=\pm\sqrt{48}$$

$$x=\pm 4\sqrt{3}$$

4. सिद्ध कीजिए कि

$$\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)^3$$

(2011, 13, 15, 19)

$$\Delta=\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$$

$$=\begin{vmatrix}a+b+c & a+b+c & a+b+c\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \quad R_1 \to R_1+R_2+R_3$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(b+c+a) & 0 \\ 2c & 0 & -(c+a+b) \end{vmatrix} \quad C_2 \to C_2-C_1,\ C_3 \to C_3-C_1$$

$$= (a+b+c)\left(+(b+c+a)(a+b+c)\right)$$

$$= (a+b+c)^3$$

5. सिद्ध कीजिए कि

$$\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix} = 4abc.$$

(2014, 15, 16, 19)

$$\Delta = \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$= \begin{vmatrix} 2(a+b) & 2(b+c) & 2(a+c) \\ a+b & b & a \\ b & b+c & c \end{vmatrix} \quad R_1 \to R_1+R_2+R_3$$

$$= 2\begin{vmatrix} a+b & b+c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & c & c \\ a+b & & \end{vmatrix} \quad R_1 \to R_1-R_2$$

$$= 2\begin{vmatrix} 0 & c & c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 0 & c \\ a+b & b-a & a \\ b & b & c \end{vmatrix} \quad C_2 \to C_2 - C_3$$

$$= 2c\begin{vmatrix} a+b & b-a \\ b & 0 \end{vmatrix}$$

$$= 2c\left(ab + b^2 - (b^2 - ab)\right)$$

$$= 2c(ab + b^2 - b^2 + ab)$$

$$= 4abc$$

6. यदि $x^y = e^{x-y}$ तो सिद्ध कीजिए कि

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2} \qquad (2019, 20)$$

हल

$$x^y = e^{x-y} \quad \text{---(1)}$$

दोनों तरफ log लेने पर

$$\log x^y = \log e^{x-y}$$

$$y \log x = (x-y)\log e$$

$$y \log x = x - y \quad \text{---(2)}$$

I° II

Diff w.r.to x

$$y \times \frac{1}{x} + \log x \cdot \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\log x \frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{y}{x}$$

$$\frac{dy}{dx}(\log x + 1) = \frac{x-y}{x}$$

$$\frac{dy}{dx} = \frac{x-y}{x}(1+\log x)$$

$$= \frac{y \log x}{x(1+\log x)}$$

7. मान ज्ञात कीजिए–

$$\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x}\, dx \qquad \text{(CBSE 2017)}$$

J sin x cos x

$$I = \int \frac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x} dx$$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin 2x = 2 \sin x \cdot \cos x$

$$= \int \frac{-\cos 2x}{\frac{2 \sin x \cdot \cos x}{2}} dx$$

$$= \frac{1}{2} \int \frac{-\cos 2x}{\sin 2x}$$

$$= \frac{1}{2} \int -\cot 2x \, dx$$

$2x = t$

$2dx = dt$

$dx = \frac{dt}{2}$

$$= -\int \cot t \, dt$$

$$= -\log|\sin t| + c$$

$$= -\log|\sin 2x| + c$$

8. $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$ का मान ज्ञात कीजिए। **(2010, 12, 16, 17)**

$$I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$$

माना $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$$= \int \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\sec^2\theta\, d\theta$$

$$= \int \cos^{-1}(\cos 2\theta)\sec^2\theta\, d\theta \qquad \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$= \int 2\theta \cdot \sec^2\theta\, d\theta$$

$$= 2\int \underset{I}{\theta} \cdot \underset{II}{\sec^2\theta}\, d\theta$$

$$= 2[\theta \cdot \tan\theta - \int 1 \cdot \tan\theta\, d\theta] \qquad \int \tan\theta\, d\theta = \log\sec\theta$$

$$= 2[\theta\cdot\tan\theta - \log\sec\theta] + C$$

$$= 2[\theta\cdot\tan\theta - \log\sqrt{1+\tan^2\theta}] + C$$

$$= 2[\tan^{-1}x \cdot x - \log\sqrt{1+x^2}] + C$$

9. $\int_{\pi/4}^{\pi/2} \underset{II}{\cos 2x} \underset{I}{\log \sin x}\, dx$ का मान ज्ञात कीजिए। **(2020)**

$$I = \int_{\pi/4}^{\pi/2} \underset{II}{\cos 2x} \cdot \underset{I}{\log\sin x}$$

$$= \log\sec x \int_{\pi/4}^{\pi/2} \cos 2x - \int_{\pi/4}^{\pi/2} \frac{1}{\sin x}\cdot\cos x \cdot \int_{\pi/4}^{\pi/2} \cos 2x\, dx$$

H.W

10. सिद्ध कीजिए कि--

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \frac{\pi}{4} \quad \textbf{(2019)}$$